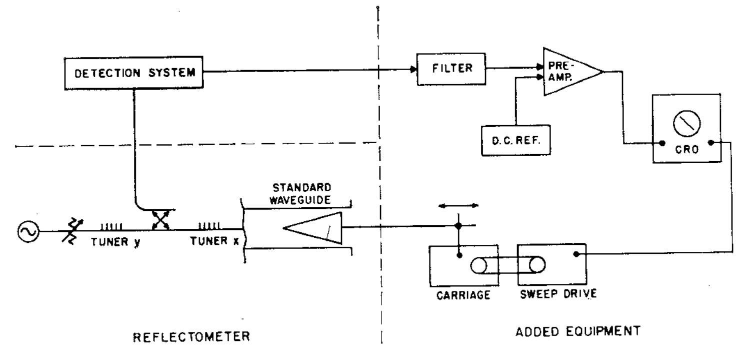


Fig. 1. Block diagram of the reflectometer system using the semi-automatic technique.

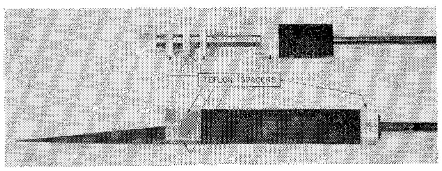


Fig. 2. Large reflection termination (upper) and low reflection termination (lower) used in tuning the reflectometer. Teflon spacers are indicated by arrows.

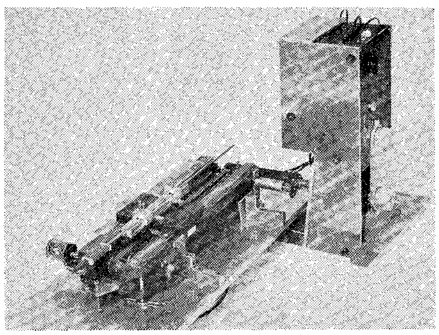

Fig. 3. "Sweep drive" unit (upper right) and carriage (lower left) used to drive the load with reciprocating longitudinal motion inside the waveguide.

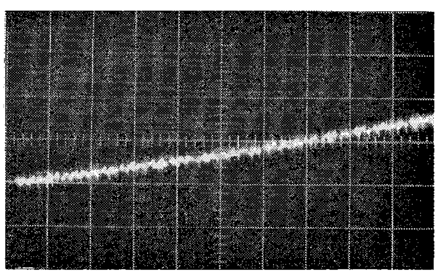

Fig. 4. Reflectometer output vs. position of the sliding short termination. Vertical sensitivity is 0.006 dB/unit and total horizontal displacement is in the neighborhood of $\lambda/2$ at 9.8 GHz.

The "sweep drive" has adjustments to control the center of the sweep, sweep speed (0.5–5 Hz), and sweep arc (30–300°). In addition, it provides the oscilloscope with a horizontal deflection voltage that is proportional to the angle (longitudinal position of load).

The output signal from the detection system should be filtered first and then applied to a "high gain dc differential pre-amplifier." It is especially convenient if this amplifier is a plug-in unit on the oscilloscope. A low-pass *RC* filter is used to restrict the ac variation of the detection system output to very low frequencies. The commercially available "high gain dc differential pre-amplifier" should have provision for use of either input, separately, or both together differentially, either ac- or dc-coupled.

The output response is centered on the oscilloscope by means of adjusting the variable dc reference voltage when the pre-amplifier is in the differential dc-coupled mode of operation.

## Brief Description of Tuning Procedure

*Preliminary Adjustment of Sweep Drive Unit*

1) The driver unit is adjusted such that the reciprocating motion of the load scans a distance of more than one-half guide wavelength. This will assure that more than one cycle of ac variation will be displayed on the oscilloscope.

2) The frequency of the driver unit is adjusted to a sweep rate of about 1 Hz (cycle per second). One uses a slow sweep rate to avoid mechanical vibration, but it must be fast enough to avoid flicker of the scope trace.

*Adjustments of the Tuners*

1) Tuner $x$ and the variable dc reference voltage are adjusted to decrease the dc level of the signal (as viewed on the scope) when a low reflection load is used and the pre-amplifier is in the differential dc-coupled mode of operation. This will assure that the directivity ratio of the reflectometer is increasing [2].

2) With the pre-amplifier switched to its ac mode and its sensitivity increased, tuner $x$ is adjusted again to minimize the ac variation as viewed on the scope. Minimum variation indicates maximum directivity ratio.

3) Tuner $y$ is adjusted next to reduce the ac variation after the low reflection load is replaced by a short.

Figure 4 shows the reflectometer output vs. position of the sliding short circuit after the system has been tuned at 9.8 GHz using this technique. The slope is an indication of the attenuation of the standard waveguide.

The average time taken for the tuning process is about ten minutes compared to a typical time of thirty minutes or more using the manual technique. In addition, this technique does not demand as much skill and knowledge of the system from the operator as the manual technique does.

## Acknowledgment

By way of acknowledgment, the author wishes to thank W. J. Foote for the design and construction of the carriage, the personnel of the Impedance projects in the Microwave Circuit Standards Section and in the Microwave Calibration Services Section for use of their systems, and his colleagues in the Microwave Circuit Standards Section for their interest and comments.

M. H. Zanboorie
National Bureau of Standards
Boulder, Colo.

## References

[1] G. F. Engen and R. W. Beatty, "Microwave reflectometer techniques," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-7, pp. 351–355, July 1959.
[2] R. W. Beatty, G. F. Engen, and W. J. Anson, "Measurement of reflections and losses of waveguide joints and connectors using microwave reflectometer techniques," *IRE Trans. on Instumentation*, vol. I-9, pp. 219–226, September 1960.
[3] W. J. Anson, "A guide to the use of the modified reflectometer technique of VSWR measurement," *Nat. Bur. Std. J. Res., Engrg. Instr.*, vol. 65C, no. 4, pp. 217–223, October–December 1961.

# A Note on Wave Propagation in Periodic Media

The solution of problems involving wave propagation in longitudinally stratified media leads to two different equations which must be solved in order to ascertain the longitudinal dependence of the field quantities [1]–[3]. These are, for the TE and TM modes respectively,

$$\frac{d^2U^{(h)}}{dz^2} + \left[\omega^2\mu_0\epsilon(z) - \gamma_h^2\right]U^{(h)} = 0 \qquad (1)$$

$$\frac{d^2U^{(e)}}{dz^2} - \frac{1}{\epsilon}\frac{d\epsilon}{dz}\frac{dU^{(e)}}{dz} + \left[\omega^2\mu_0\epsilon(z) - \gamma_e^2\right]U^{(e)} = 0 \qquad (2)$$

in which $\omega$ denotes the frequency, $\mu_0$ the (constant) permeability and $\epsilon$ the permittivity of the medium, and $\gamma^2$ the sum of the squares of the transverse separation constants. $U$ is a function describing the longitudinal dependence of a field component or of a scalar potential function from which the field quantities may be derived, and $z$ denotes the longitudinal coordinate.

It will be shown that when $\epsilon$ is a continuous even-periodic function of $z$ with period $p$ the solutions to (1) and (2) may be expressed in terms of solutions to Hill's equation [4], the method of solution of which is tedious but straightforward.

One makes the substitutions

$$\zeta = \frac{\pi z}{p} \qquad (3)$$

$$U^{(h)}(z) = f^{(h)}(\zeta) \qquad (4)$$

in (1) yielding

$$\frac{d^2f^{(h)}}{d\zeta^2} + \left(\frac{p}{\pi}\right)^2(\omega^2\mu_0\epsilon - \gamma^2)f^{(h)} = 0. \qquad (5)$$

Manuscript received April 26, 1965.

If the function

$$\lambda(\zeta) = \left(\frac{p}{\pi}\right)^2 (\omega^2\mu_0\epsilon - \gamma^2) \quad (6)$$

is expansible in an absolutely convergent Fourier cosine series [5]

$$\lambda(\zeta) = \theta_0^{(h)} + 2\sum_{n=1}^{\infty} \theta_n^{(h)} \cos 2n\zeta \quad (7)$$

(5) becomes

$$\frac{d^2f^{(h)}}{d\zeta^2} + \left(\theta_0^{(h)} + 2\sum_{n=1}^{\infty}\theta_n^{(h)}\cos 2n\zeta\right) f^{(h)} = 0 \quad (8)$$

which is the canonical form of Hill's equation. The computation of stability charts for Hill's equation, and, thus, the determination of the pass band and stop band structure of the dispersion characteristics, follows from the characteristic equation for Hill's equation

$$\sin^2\frac{\pi\beta}{2} = \Delta^{(h)}(0)\sin^2\frac{\pi\sqrt{\theta_0^{(h)}}}{2} \quad (9)$$

where $\beta$ denotes the propagation factor in the Floquet solution to Hill's equation and $\Delta^{(h)}(0)$ is an infinite determinant whose elements are

$$\Delta^{(h)}(0)\,|_{mm} = 1 \quad (10)$$

$$\Delta^{(h)}(0)\,|_{mn} = \frac{\theta^{(h)}m - n}{\theta_0^{(h)} - 4m^2} \; (m \neq n). \quad (11)$$

This procedure has been described in [1] and [3]. The solution to Hill's equation may be obtained, when $\beta$ is known, by means of a procedure outlined in [5] and [6]

In the case of TM wave propagation, one introduces into (2) the substitutions

$$\zeta = \frac{\pi z}{p} \quad (12)$$

$$U^{(e)}(z) = \epsilon^{1/2}f^{(e)}(\zeta) \quad (13)$$

yielding the differential equation for $f^{(e)}(\zeta)$,

$$\frac{d^2f^{(e)}}{d\zeta^2} + \left[\frac{1}{2\epsilon}\frac{d^2\epsilon}{d\zeta^2} - \frac{3}{4\epsilon^2}\left(\frac{d\epsilon}{d\zeta}\right)^2 + \left(\frac{p}{\pi}\right)^2(\omega^2\mu_0\epsilon - \gamma^2)\right] f^{(e)} = 0. \quad (14)$$

If $\epsilon$ is an even-periodic function, so also is the function in square brackets and, thus, one may write

$$[\quad] = \theta_0^{(e)} + 2\sum_{n=1}^{\infty}\theta_n^{(e)}\cos 2n\zeta \quad (15)$$

if the series is absolutely convergent. Hence,

$$\frac{d^2f^{(e)}}{d\zeta^2} + \left[\theta_0^{(e)} + 2\sum_{n=1}^{\infty}\theta_n^{(e)}\cos 2n\zeta\right] f^{(e)} = 0 \quad (16)$$

which is again the canonical form of Hill's equation.

Thus, the $z$ dependence of both TE and TM waves in periodic media is expressible in terms of Hill functions. The pass band and stop band characteristics or $\omega$-$\beta$ diagrams may be determined from the characteristic equation by numerical or graphical methods and the functional dependence of the fields from the solutions to the Hill equation.

K. F. Casey
Air Force Institute of Technology
Wright-Patterson AFB
Dayton, Ohio

References

[1] T. Tamir, H. C. Wang, and A. A. Oliner, "Wave propagation in sinusoidally stratified dielectric media," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 323–335, May 1964.
[2] C. Yeh and Z. A. Kaprielian, "On Inhomogeneously Filled Waveguides," Elec. Engrg. Dept., University of Southern California, USCEC Rept. 84–206, 1963.
[3] C. Yeh, K. F. Casey, and Z. A. Kaprielian, "Transverse magnetic wave propagation in sinusoidally stratified dielectric media," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-13, pp. 297–302, May 1965.
[4] G. W. Hill, "Mean Motion of the Lunar Peigee," *Acta. Math.*, vol. VIII, 1886.
[5] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, New York: Cambridge University Press, 1963.
[6] N. W. McLachlan, *Theory and Application of Mathieu Functions*, New York: Clarendon Press, 1947.

## Note on the Measurement of Material Properties by the Strip-Line Cavity

It has been found that when making measurements of the properties of materials with a strip-line cavity [1]–[3], results are obtained which are consistently lower than expected. The error is the more serious, the higher the value of the dielectric constant or permeability, as the case may be, of the sample.

In the case of measurements of magnetic properties, the reason for the effect has been explained elsewhere [4]. The discrepancy is attributable to demagnetizing factors in the specimen, and when this is in the form of a flat slab, placed either vertically against the end wall of the cavity or horizontally on the strip, the true relative permeability of an isotropic specimen $\bar{\mu}$ is given by

$$\bar{\mu} = \frac{\mu(1 - N)}{1 - \mu N} \quad (1)$$

where $\mu$ is the apparent permeability given by the perturbation formulae of [1] and [3], and $N$ is the demagnetizing factor of the specimen appropriate to the direction of the microwave magnetic field (in MKS units). It is apparent from this formula that the difference between $\mu$ and $\bar{\mu}$ increases with increasing $\mu$ and $\bar{\mu}$, being zero for $\mu = \bar{\mu} = 1$. For large values of $\bar{\mu}$, $\mu$ approaches the limiting value $1/N$.

In the case of measurements of the dielectric constant, the discrepancy is attributable to the presence of minute air gaps between the specimen and the strip and ground plane. The perturbation formula of [1] and [3] gives a value for the dielectric constant $\epsilon$ which would be correct if the sample fitted flush with the strip and ground plane. If there is an appreciable gap, the value of $\epsilon$ calculated from the perturbation formula is an apparent one. The relation between $\epsilon$ and the true dielectric constant $\bar{\epsilon}$ can be calculated by means of the concept of a dielectric circuit analogous to the well-known magnetic circuit. The result is

$$\bar{\epsilon} = \frac{\epsilon(1 - x/h)}{1 - \epsilon x/h} \quad (2)$$

where $x$ is the total gap height, i.e., the sum of the gaps at top and bottom of the sample, and $h$ is the distance from the strip to the ground plane, i.e., the distance $b-t$ in the notation of [1]–[3]. It is apparent from this formula that the difference between $\epsilon$ and $\bar{\epsilon}$ increases with increasing $\epsilon$ and $\bar{\epsilon}$, being zero for $\epsilon = \bar{\epsilon} = 1$. For large values of $\bar{\epsilon}$, $\epsilon$ approaches the limiting value $h/x$.

The authors wish to thank the Director of Research of The Marconi Company for permission to publish this note.

R. A. Waldron
S. P. Maxwell
Research Div.
The Marconi Co. Ltd.,
Great Baddow, Essex, England

References

[1] R. A. Waldron, "Theory of a strip-line cavity for measurement of dielectric constants and gyromagnetic-resonance line-widths," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 123–131, January 1964.
[2] S. P. Maxwell, "A strip-line cavity resonator for measurement of magnetic and dielectric properties of ferrites at low microwave frequencies," *Marconi Rev.*, vol. 27, pp. 22–29, 1st Quarter, 1964.
[3] R. A. Waldon, "Theory of the strip-line cavity resonator," *ibid.*, pp. 30–42.
[4] S. P. Maxwell, "A strip-line cavity resonator for measurement of magnetic and dielectric properties of ferrites at low microwave frequencies," *Microwave J.*, to be published.

Manuscript received May 3, 1965.

## On Mode Losses in Confocal Resonator and Transmission Systems

In a recent correspondence Lonngren and Beyer [1] calculated the losses for a single iteration in a "beam waveguide" [2] with circular lenses separated by twice their focal length. Since the confocal Fabry-Perot resonator with two identical circular mirrors may be studied by superimposing two guided wave beams propagating oppositely in the given system, the beam-waveguide losses allow one to determine the resonator $Q$. The problem which Lonngren and Beyer [1] have solved approximately is to find the eigenvalues $\gamma_{\alpha,n}(c)$ of the integral equation

$$\gamma_{\alpha,n}(c)\mathcal{S}_{\alpha,n}(c, x) = \int_0^1 cJ_\alpha(cxy)\mathcal{S}_{\alpha,n}(c, y)y\,dy \quad (1)$$

for small $c$. In an earlier work Beyer and Scheibe [3] obtained values of $\gamma_{\alpha,n}(c)$ for large $c$. The purpose of this correspondence is to point out that the same information has been obtained by directly studying the solutions of (1).

Manuscript received May 17, 1965.